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Confining strings from G_2-holonomy spacetimes.

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Abstract

The low energy physics of M theory near certain singularities of G_2-holonomy spaces can be described by pure $\mathcal{N} = 1$ super Yang-Mills theory in four dimensions. In this note we consider the cases when the gauge group is $SO(2n)$, E_6, E_7 or E_8. Confining strings with precisely the expected charges are naturally identified in proposed "gravity duals" of these singular M theory spacetimes.

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In [1] Vafa described a duality between the wrapped D6-brane system on $T^*(S^3)$ - ie the *deformation* of the conifold singularity in three complex dimensions - and the closed Type IIA string backround which is the *resolution* of the same conifold singularity plus Ramond-Ramond fluxes. More precisely the duality relates open string amplitudes on the D6-brane side to closed string amplitudes on the closed string side.

In [2] we showed that both of the Type IIA string backgrounds involved in this duality arise from G_2-holonomy vacua of M theory. In [3], it was proposed that the two sides of this story are continuously connected in the one complex dimensional moduli space - the two topologically distinct 7-manifolds being related by a flop transition. On the D6-brane side the corresponding 7-manifold is a singular orbifold of the total space $S(S^3)$ of the spin bundle over S^3 which is well known to admit a G_2-holonomy metric [4]. If the IIA background has n D6-branes then the singularities are a family of A_{n-1} orbifold singularities in $\mathbb{R}^4$ fibered over the S^3 which is the zero section of $S(S^3)$. On the closed string side the 7-manifold is simply $S(S^3/\mathbb{Z}_n)$ - the standard spin bundle over the Lens space $S^3/\mathbb{Z}_n$ - also a smooth G_2-holonomy manifold.

A natural question to consider is whether or not this kind of duality extends to other gauge groups? One can easily replace the S^3 family of A_{n-1} orbifold singularities by D_k, E_6, E_7 or E_8 singularities. The "gravity duals" of M theory on these singular G_2-holonomy spaces would then naturally be given by M theory on $S(S^3/\Gamma)$, with Γ the corresponding D_k or E_i type subgroup of $SU(2)$. For the D_k cases this has recently been studied in [5] where several positive checks of the duality were made.

At low energies below the scale set by the S^3, the physics of M theory on a G_2-holonomy, S^3 family of A, D or E singularities is described by $\mathcal{N} = 1$ super Yang-Mills theory in four dimensions with A, D or E gauge group. These gauge theories are expected to confine at low energies. A test of a dual description of these M theory backgrounds - as M theory on the smooth G_2-holonomy spaces $S(S^3/\Gamma)$ - would be to identify the confining strings. That is the purpose of this note.

Topologically, $S(S^3/\Gamma)$ is equivalent to $\mathbb{R}^4 \times S^3/\Gamma$. Strings in four dimensions can be formed by wrapping $M2$-branes on one-cycles or $M5$-branes on four-cycles. In this case, the 7-manifold has one-cycles but no four-cycles, so the natural candidates for confining strings are $M2$-branes wrapping one-cycles in S^3/Γ.

The charges of the strings obtained by wrapping the $M2$-branes are given by $H_1(S(S^3/\Gamma)) \cong H_1(S^3/\Gamma) \cong \Gamma/[\Gamma,\Gamma]$. In the second isomorphism we use the fact that the first homology group of a manifold is isomorphic to the

abelianisation[2] of its fundamental group. The fundamental group of the manifold under consideration is simply Γ.

So, in order to determine the charges of our candidate strings we need to compute the abelianisations of all of the finite subgroups of $SU(2)$.

For $\Gamma \cong \mathbb{Z}_n$, the gauge group is locally $SU(n)$. Since $\mathbb{Z}_n$ is abelian, its commutator subgroup is trivial and hence the charges of our strings take values in $\mathbb{Z}_n$. Since this is the center of $SU(n)$ this is the expected answer for a confining $SU(n)$ theory.

For $\Gamma \cong \mathbb{D}_{k-2}$, the binary dihedral group of order $4k-8$, the local gauge group of the Yang-Mills theory is $SO(2k)$. The binary dihedral group is generated by two elements α and β which obey the relations

$$\alpha^2 = \beta^{k-2} \tag{1}$$

$$\alpha\beta = \beta^{-1}\alpha \tag{2}$$

$$\alpha^4 = \beta^{2k-4} = 1 \tag{3}$$

To compute the abelianisation of $\mathbb{D}_{k-2}$, we simply take these relations and impose that the commutators are trivial. From the second relation this implies that

$$\beta = \beta^{-1} \tag{4}$$

which in turn implies that

$$\alpha^2 = 1 \quad for \quad k = 2p \tag{5}$$

and

$$\alpha^2 = \beta \quad for \quad k = 2p+1 \tag{6}$$

Thus, for $k = 2p$ we learn that the abelianisation of $\mathbb{D}_{k-2}$ is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$, whereas for $k = 2p+1$ it is isomorphic to $\mathbb{Z}_4$. These groups are respectively the centers of $Spin(4p)$ and $Spin(4p+2)$. This is the expected answer for the confining strings in $SO(2k)$ super Yang-Mills which can be coupled to spinorial charges.

To compute the abelianisations of the binary tetrahedral (denoted $\mathcal{T}$), octahedral ($\mathcal{O}$) and icosahedral ($\mathcal{I}$) groups which correspond respectively to E_6, E_7 and E_8 super Yang-Mills theory, we utilise the fact that the order of $G/[G,G]$ - with G a finite group - is the number of inequivalent one dimensional representations of G. The representation theory of the finite subgroups of $SU(2)$ is described through the Mckay correspondence by the

[2]The abelianisation of a finite group is its quotient by the group generated by all commutators. This group also plays a crucial role in classifying bound states of D-branes wrapping submanifolds with non-trivial fundamental group [6].

representation theory of the corresponding Lie algebras. In particular the dimensions of the irreducible representations of $\mathcal{T}, \mathcal{O}$ and $\mathcal{I}$ are given by the coroot integers (or dual Kac labels) of the affine Lie algebrae associated to E_6, E_7 or E_8 respectively. From this we learn that the respective orders of $\mathcal{T}/[\mathcal{T},\mathcal{T}]$, $\mathcal{O}/[\mathcal{O},\mathcal{O}]$ and $\mathcal{I}/[\mathcal{I},\mathcal{I}]$ are three, two and one. Moreover, one can easily check that $\mathcal{T}/[\mathcal{T},\mathcal{T}]$ and $\mathcal{O}/[\mathcal{O},\mathcal{O}]$ are $\mathbb{Z}_3$ and $\mathbb{Z}_2$ respectively, by examining their group relations. Thus we learn that $\mathcal{T}/[\mathcal{T},\mathcal{T}]$, $\mathcal{O}/[\mathcal{O},\mathcal{O}]$ and $\mathcal{I}/[\mathcal{I},\mathcal{I}]$ are, respectively isomorphic to the centers $Z(E_6)$, $Z(E_7)$ and $Z(E_8)$ in perfect agreement with the expectation that the super Yang-Mills theory confines.

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References

[1] C. Vafa, Superstrings and Topological Strings at Large n, hep-th/0008142.

[2] B.S. Acharya, On Realising $\mathcal{N}=1$ super Yang-Mills in M theory, hep-th/0011089.

[3] M. Atiyah, J. Maldacena, and C. Vafa, An M theory Flop as a Large n Duality, hep-th/0011256.

[4] R. Bryant and S. Salamon, On the Construction of Some Complete Metrics With Exceptional Holonomy, Duke Math Journal **58** 3 (1989) 829.

[5] S. Sinha and C. Vafa, SO and Sp Chern-Simons at Large n, hep-th/0012136.

[6] R. Gopakumar and C. Vafa, Branes and Fundamental Groups, ATMP 2 (1998) 399. hep-th/9712048.